

**ANNA UNIVERSITY COIMBATORE**  
**B.E. / B.Tech. DEGREE EXAMINATION – DECEMBER 2008**  
**THIRD SEMESTER**  
**(Common to EEE / ECE / EIE / ICE / MECHATRONICS /**  
**TEXTILE TECH(FT) / TEXTILE TECH. / MEDICAL ELECTRONICS)**

**SM 302 - ENGINEERING MATHEMATICS – III**

**Time: Three Hours**

**Maximum: 100 Marks**

**PART A – (20 x 2 = 40 Marks)**

**Answer ALL Questions**

1) Write down the Dirichlet's condition for a function to be expanded as a Fourier series.

2) Define the value of the Fourier series of  $f(x)$  at a point of discontinuity.

3) If  $f(x) = \sin hx$  is defined in  $-\pi < x < \pi$ , write the values of Fourier coefficients  $a_0$  and  $a_n$ .

4) If  $x = 2 \left[ \frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \dots \right]$  in  $0 < x < \pi$ , Prove that

$$\sum \frac{1}{n^2} = \frac{\pi^2}{6}.$$

5) Prove that if  $F\{f(x)\} = F(s)$ , then  $F\{f(x-a)\} = e^{isa} F(s)$ .

6) Find the Fourier transform of  $f(x)$  defined by  $f(x) = \begin{cases} 1, & \text{if } a < x < b \\ 0, & \text{otherwise} \end{cases}$ .

7) Find the Fourier Cosine transform of.  $f(x) = \begin{cases} x, & 0 < x < \pi \\ 0, & x \geq \pi \end{cases}$

8) If  $F\{f(x)\} = F(s)$ , prove that  $F\{x^2 f(x)\} = -\frac{d^2}{ds^2} F(s)$ .

9) State Initial and Final value theorem on Z - transform.

10) Find the Z – transform of  $\frac{1}{(n+1)}$  .

11) Prove that  $Z^{-1} \left[ \frac{z^2}{(z-a)^2} \right] = (n+1) a^n$  .

12) Find the difference equation from  $y(n) = (A + n B) 2^n$  .

13) Form the partial differential equation by eliminating the arbitrary constants

‘a’ and ‘b’ from  $z = a x^3 + b y^3$  .

14) Find the Singular solution of  $z = px + qy + p^2 + q^2 + 1$  .

15) Find the general solution of  $px + qy = z$

16) Find the particular integral of  $(D^2 - 4DD')z = e^{3x+4y}$

17) Classify the p.d.e  $(1+x^2)(4+x^2)u_{xx} + (5+2x^2)u_{xy} + u_{yy} = 0$

18) Write any two assumptions made while deriving the partial differential equation of transverse vibrations of a string.

19) Define steady state. Write the one dimensional heat equation in steady state.

20) Write all the solutions of Laplace equation in Cartesian form, using the method of separation of variables

**PART B – (5 x 12 = 60 Marks)**

**Answer Any FIVE Questions**

- 21) a) Find the Fourier series expansion for  $f(x) = x^2$  in  $(-\pi, \pi)$  and hence show

that 
$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty = \frac{\pi^4}{90} \quad (8)$$

- b) Obtain the half range cosine series for  $f(x) = (x - 2)^2$  in the interval  $(0, 2)$  (4)

- 22) Find the Fourier transform of  $f(x) = \begin{cases} 1 - |x| & \text{for } |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$

Hence find the values of (i)  $\int_0^\infty \frac{\sin^2 t}{t^2} dt$  and (ii)  $\int_0^\infty \frac{\sin^4 t}{t^4} dt$  (12)

- 23) a) Using convolution theorem evaluate the inverse Z-transform of  $\frac{z^2}{(z-1)(z-3)}$ . (6)

b) Find the inverse Z- transform of  $\frac{z^3}{(z-1)^2(z-2)}$ . (6)

- 24) a) Find the Z- transform of  $(n+1)(n+2)$  (4)

- b) Using Z-transforms, solve  $y(n+2) + 3y(n+1) - 4y(n) = 0$ ,  $n \geq 1$ , given that

$y(0) = 3$  and  $y(1) = -2$ . (8)

25) a) Obtain the complete solution of the equation  $z = px + qy - 2\sqrt{pq}$  (6)

b) Solve  $(D^2 + DD' - 6D'^2)z = \cos(2x+y)$  (6)

26) a) Solve  $xz p + yz q = xy$ . (6)

b) Solve  $(2D^2 - 5DD' + 2D'^2)z = e^{2x+y}$ . (6)

27) A tightly stretched flexible string has its ends fixed at  $x = 0$  and  $x = l$ . At time  $t = 0$ , the string is given a shape defined by  $f(x) = kx(l - x)$ , where  $k$  is a constant, and then released from rest. Find the displacement of any point  $x$  of the string at any time  $t$ . (12)

28) An infinitely long rectangular plate with insulated surfaces is 10cm wide. The two long edges and one short edge are kept at zero temperature, while the other short edge  $x = 0$  is kept at temperature given by

$$u = \begin{cases} 20y & \text{for } 0 \leq y \leq 5 \\ 20(10 - y) & \text{for } 5 \leq y \leq 10 \end{cases}$$

Find the steady state temperature at any point in the plate. (12)

\*\*\*\*\*THE END\*\*\*\*\*